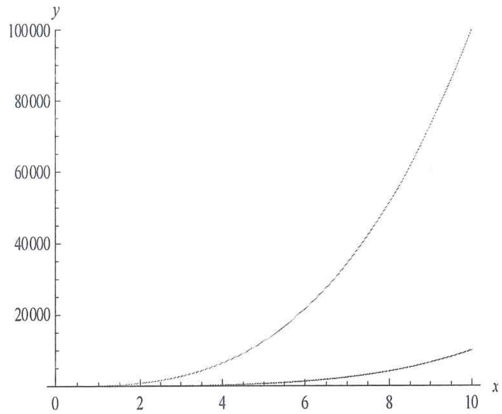
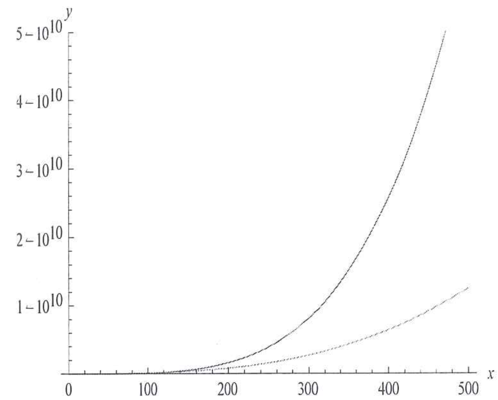
Sec. 11.6 Comparing Power, Exponential and Log Functions

Comparing Power Functions:
When comparing power functions with positive coefficients, higher powers dominate.

Ex. Let $f(x) = 100x^3$ and $g(x) = x^4$ for $x > 0$. Compare the long-term behavior of these two functions using graphs.



Close-up View

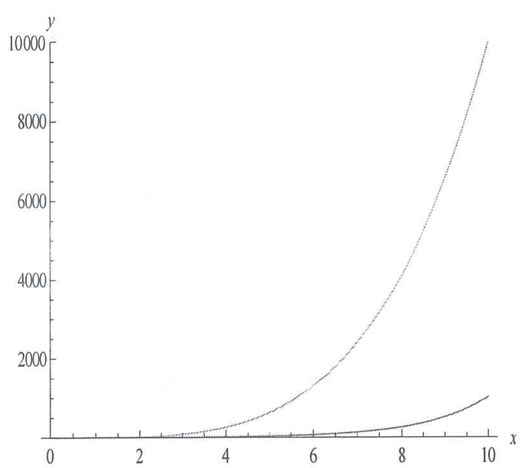


Far-Away View

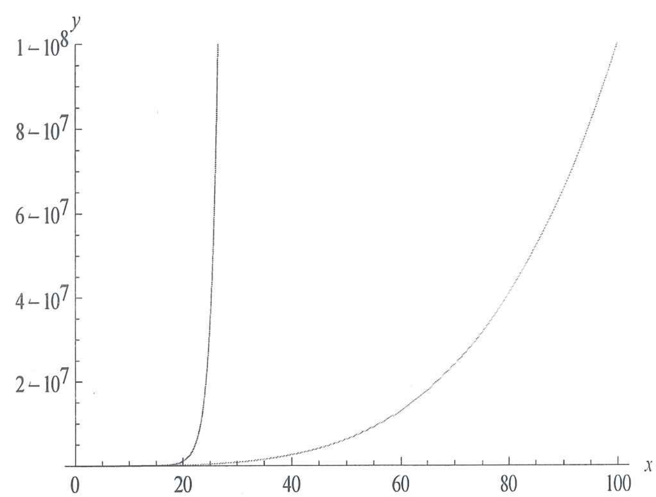
Comparing Exponential and Power Functions:

Any positive increasing exponential function eventually grows faster than any power function.

Ex. Compare the functions $f(x) = x^4$ and $g(x) = 2^x$ for $x > 0$.



Close-up View

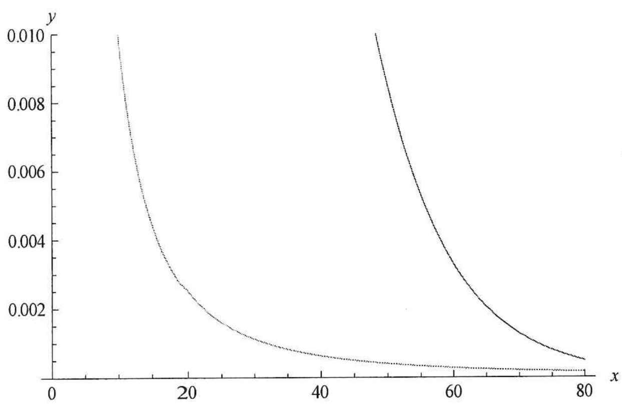


Far-Away View

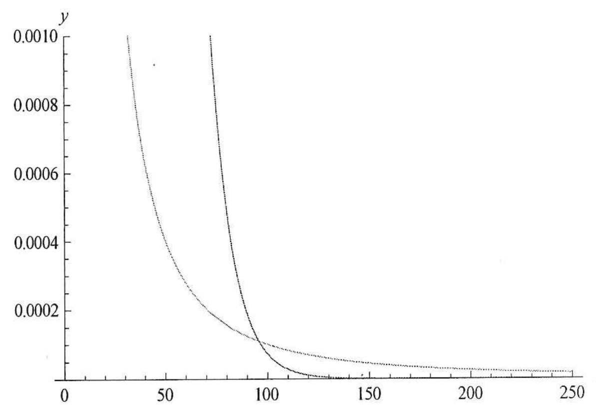
Decreasing Exponential and Decreasing Power Functions:

Any positive decreasing exponential function eventually approaches the horizontal axis faster than any positive decreasing power function.

Ex Compare the functions $f(x) = x^{-2}$ and $g(x) = 1.1^{-x}$ for $x > 0$.



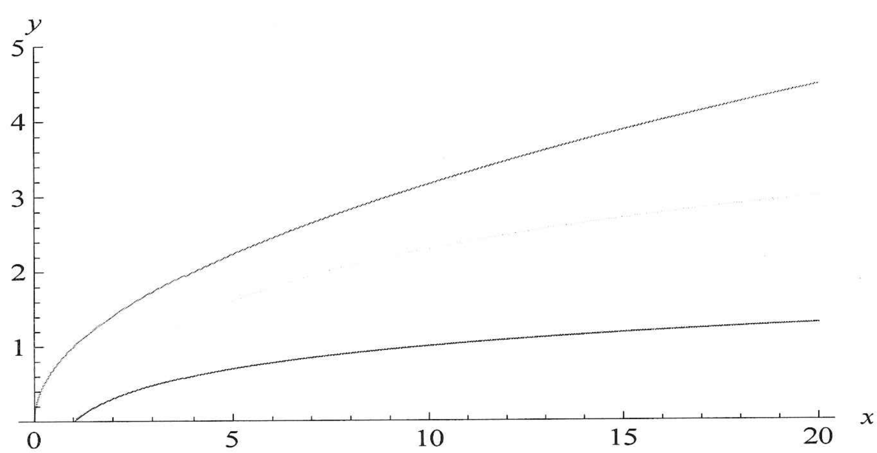
Close-up View



Far-away View

Comparing Log and Power Functions:
Any positive increasing power function eventually grows more rapidly than y = log x and y = ln x.

Ex. Compare the functions $f(x) = x^{1/2}$, $g(x) = \log x$, and $h(x) = \ln x$ for $x > 0$.



Ex: Find a possible formula for f if f is a linear, exponential, and a power function if $f(1) = 18$ and $f(3) = 1458$.

(1, 18) (3, 1458)

LINEAR: $m = \frac{1458-18}{3-1}$

$m = \frac{1440}{2}$

$m = 720$

$y = mx + b$

$18 = 720(1) + b$

$18 = 720 + b$

$-702 = b$

$f(x) = 720x - 702$

EXPONENTIAL: $y = ab^x$

$\frac{y = ab^x}{y = ab^x}$

$\frac{1458 = ab^3}{18 = ab^1}$

$81 = b^2$

$9 = b$

$y = ab^x$

$18 = a(9)^1$

$18 = 9a$

$2 = a$

$f(x) = 2(9)^x$

POWER: $\frac{y = kx^p}{y = kx^p}$

$\frac{1458 = k(3)^p}{18 = k(1)^p}$

$81 = 3^p \longrightarrow p = 4$

or

$\log 81 = \log(3^p)$

$\frac{\log 81}{\log 3} = \frac{p \cdot \log 3}{\log 3}$

$4 = p$

$y = Kx^p$

$18 = K(1)^4$

$18 = K \cdot 1$

$18 = K$

$f(x) = 18x^4$

HW: pg 472 -474 #1-14,16,18,25,27,28,31,35,37